1. Below is a list of inference rules for functional dependencies. Some are valid and some are not. Using inference rules IR1 through IR6 from Section 10.2.2 of Elmasri & Navathe, prove at least two of the valid rules given here. (Your proofs should be of the same form as the proofs of IR4, IR5, and IR6 in Section 10.2.2.)

Then choose at least two of the invalid rules given here and show that they are invalid. Do this by spelling out, for each such rule, a relation instance/state that is consistent with the functional dependencies on the rule’s left-hand side but not consistent with those on the rule’s right-hand side. As an example, consider the (obviously invalid) inference rule \( \{X \rightarrow Y\} \models \{Y \rightarrow X\} \). The following relation state is consistent with the left-hand side but inconsistent with the right-hand side; hence, the inference rule is not valid.

\[
\begin{array}{cc}
X & Y \\
x_0 & y_0 \\
x_1 & y_0 \\
\end{array}
\]

Here we have followed the convention that \( x_i \) (for natural number \( i \)) is a value in the domain of attribute \( X \) and that \( i \neq j \) implies \( x_i \neq x_j \). And similarly for \( Y \). We strongly recommend that you follow the same convention in your answers.

Choosing inference rules (k) and (l) will gain you a little extra credit.

(a) \( \{W \rightarrow Y, X \rightarrow Z\} \models \{WX \rightarrow Y\} \)
(b) \( \{X \rightarrow Y\} \text{ and } Z \subseteq Y \models \{X \rightarrow Z\} \)
(c) \( \{X \rightarrow Y, X \rightarrow W, WY \rightarrow Z\} \models \{X \rightarrow Z\} \)
(d) \( \{XY \rightarrow Z, Y \rightarrow W\} \models \{XW \rightarrow Z\} \)
(e) \( \{X \rightarrow Z, Y \rightarrow Z\} \models \{X \rightarrow Y\} \)
(f) \( \{X \rightarrow Y, XY \rightarrow Z\} \models \{X \rightarrow Z\} \)
(g) \( \{X \rightarrow Y, Z \rightarrow W\} \models \{XZ \rightarrow YW\} \)
(h) \( \{XY \rightarrow Z, Z \rightarrow X\} \models \{Z \rightarrow Y\} \)
(i) \( \{X \rightarrow Y, Y \rightarrow Z\} \models \{X \rightarrow YZ\} \)
(j) \( \{XY \rightarrow Z, Z \rightarrow W\} \models \{X \rightarrow W\} \)
(k) \( \{X \rightarrow W, Y \rightarrow Z, WYZ \rightarrow U, WZ \rightarrow V, V \rightarrow X\} \models \{XY \rightarrow UV\} \)
(l) \( \{X \rightarrow W, Y \rightarrow Z, WYZ \rightarrow U, WZ \rightarrow V, V \rightarrow X\} \models \{XZ \rightarrow Y\} \)
2. Determine whether $F$ and $G$ are equivalent (as sets of functional dependencies go), where

$$F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\} \quad \text{and} \quad G = \{A \rightarrow CD, E \rightarrow AH\}$$

The meaning of “equivalent” in this context is given in Section 10.2.3 of Elmasri & Navathe. Note that Algorithm 10.1 is a valuable tool for answering this question.

Your answer should explain what steps you took (and the partial results obtained thereby) in arriving at your conclusion.

3. Suppose that the relation $R(A, B, C, D, E, F, G, H, I)$ has the following functional dependencies:

- $FD_1: \{A, B\} \rightarrow \{C, D\}$
- $FD_2: \{A\} \rightarrow \{E, F, H\}$
- $FD_3: \{B\} \rightarrow \{G, I\}$
- $FD_4: \{E, F\} \rightarrow \{H\}$
- $FD_5: \{G\} \rightarrow \{I\}$

Decompose $R$ into 3NF relations. (You may wish to decompose them into 2NF relations along the way.)

4. Consider the relation $R$ from the previous problem, and suppose that it also had the functional dependency

$$FD_6: \{C\} \rightarrow \{B, D\}$$

With this new functional dependency, we see that $R$ has a second candidate key, $\{A, C\}$. Using the “general” definitions of 2NF and 3NF from Section 10.4 of Elmasri & Navathe, decompose $R$ into 3NF relations.

Are all the resulting relations in BCNF (Boyce-Codd Normal Form)? If not, point out any “offending” relation.