MATH 103 Pre-Calculus Mathematics
Quiz #9 Fall 2008
Sample Solutions

1. Consider the rational function

$$f(x) = \frac{3(x+2)(x-1)(x-3)}{(x+2)^2(x-1)}$$

(a) Write f in "simplest" form.
Solution: Cancelling factors common to the numerator and denominator, we get

$$f(x) = \frac{3(x-3)}{x+2} \quad (\text{for } x \neq 1)$$

(b) What is its domain?
Solution: f is undefined for $x = -2, 1$ (because those are the values of x making its denominator zero), which leaves as its domain $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

(c) What (if any) are its x-intercepts (i.e., zeros)?
Solution: f has as its lone zero $x = 3$, which is to say that its only x-intercept occurs at $(3, 0)$. This follows from the fact that, in its simplified form, the only value of x making f's numerator zero is 3.

Note that neither $x = -2$ nor $x = 1$ is a zero of f, because neither is in f's domain, which is to say that neither $f(-2)$ nor $f(1)$ is defined. (Recall that for $x = a$ to be a zero of f means that $f(a) = 0$, which is inconsistent with $f(a)$ being undefined.)

(d) What (if any) are its horizontal asymptotes?
Solution: $y = 3$ is f's horizontal asymptote. This follows from the fact that f's numerator and denominator are of the same degree and the ratio of their leading coefficients is 3.

(e) What (if any) are its vertical asymptotes?
Solution: $x = -2$ is f's lone vertical asymptote. This follows from the fact that, in its simpified form, the only value of x making f's denominator zero is -2.

2. Determine a rational function g having vertical asymptotes at $x = -3$ and $x = 2$, a horizontal asymptote at $y = 3$, and a lone x-intercept (i.e., zero) at $x = 1$.

Solution: For g to have vertical asymptotes at $x = -3$ and $x = 2$ indicates that $x + 3$ and $x - 2$ are factors of its denominator. For g to have a zero at $x = 1$ indicates that $x - 1$ is a factor of its numerator.

So far we have as a rough solution

$$g(x) = \frac{x - 1}{(x + 3)(x - 2)}$$

For g to have $y = 3$ as a horizontal asymptote indicates not only that its numerator and denominator are of the same degree but also that the leading coefficient of its numerator is three times that of its denominator.

To achieve the first of these two conditions, we must insert a linear factor (i.e., one of the form $x - a$) into the numerator. But we don't want to introduce a new zero (i.e., one other than $x = 1$, which the problem statement identified as the "lone" zero), so we should choose $a = 1$ (and thereby make $x = 1$ a zero of multiplicity two).

To achieve the second condition, we should insert the constant factor 3 into the numerator. The result is

$$g(x) = \frac{3(x - 1)^2}{(x + 3)(x - 2)}$$