1. In class, we developed a program for solving the two-color version of the Dutch National Flag problem. The program was developed using a proposed loop invariant (derived from the post-condition) as a guide.

Here you are asked to develop an alternative solution to the same problem. As in class, your program is not to modify the array except by swapping array elements. (Assume that there is a method swap such that the effect of making the call swap(a, k, j) is to swap the values in a[k] and a[j].)

The pre-condition is that every element in the array a[] satisfies exactly one among the two predicates isRed() and isBlue(). The postcondition is as indicated in this picture:

```
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>k</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>a</td>
<td>all RED</td>
<td>all BLUE</td>
<td>&amp;&amp; 0&lt;=k&lt;=N</td>
</tr>
</tbody>
</table>
```

(We use N as an abbreviation for the length of a, which in Java is written a.length.) In words, this says that 0 ≤ k ≤ N and that every element in the array segment a[0..k − 1] is Red and every element in a[k..N − 1] is Blue.

More formally, we could express this in the language of predicate logic as follows:

\((\forall i \mid 0 \leq i < k : isRed(a[i])) \land (\forall i \mid k \leq i < N : isBlue(a[i])) \land 0 \leq k \leq N\)

The loop invariant of your program should be as suggested by this picture:

```
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>k</th>
<th>m</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>a</td>
<td>all RED</td>
<td>?</td>
<td>all BLUE</td>
<td>&amp;&amp; 0&lt;=k&lt;=m&lt;=N</td>
</tr>
</tbody>
</table>
```

In words, this says that 0 ≤ k ≤ m ≤ N and that every element in a[0..k − 1] is Red and every element in a[m..N − 1] is Blue. More formally, we can express this by

\((\forall i \mid 0 \leq i < k : isRed(a[i])) \land (\forall i \mid m \leq i < N : isBlue(a[i])) \land 0 \leq k \leq m \leq N\)

Arrive at your solution by correctly replacing each question mark in the following with either an expression or a sequence of statements, whichever is appropriate. The initialization of k and m should establish the invariant by making the ?-region cover the entire array. Each iteration of the loop should decrease by at least one the length of the ?-region.

1
k = ?;  m = ?;

while ( ? ) {
    if (isRed(a[k]))
        { ? }
    else /* isBlue(a[k]) */
        { ? }
}